

**MEASURING PRODUCTIVITY**  
**FROM VERTICALLY INTEGRATED SECTORS.**

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***ABSTRACT***

There are many ways to measure productivity. The final judgment is going to depend on the suitability of each index to the main purpose the researcher has in mind. Whenever we are interested in "competitiveness", the proper measure will be the inverse of the total labor embodied in one unit of final product; or, what amounts to the same, the labor employed in the *vertically integrated sectors* corresponding to each final product. A weighted mean of this sectoral measure yields an index of aggregate productivity suitable to measure the potential welfare and the potential growth of an economy. Under this perspective -that can be called Sraffian or Ricardian- we shall make a critical review to the conventional measures of productivity based either on "direct labor", or on "total factors of production". Finally, we shall compare the different indexes using data from the Spanish economy in the period 1970-88.

# MEASURING PRODUCTIVITY

## FROM VERTICALLY INTEGRATED SECTORS.

### I. INTRODUCTION.

In recent years, the measure of productivity has occupied a substantial part of economic research, both theoretical and applied. Any index of productivity entails a ratio between “output” and “inputs”. The difficulties and disagreements come up at the moment of deciding what should be included in the numerator and in the denominator. Our paper will concern with this “basic” issue. We shall forget more concrete problems such as the selection of deflators, the homogenization of output in the presence of qualitative changes, or the smoothing out of cyclical components which affect the degree of utilization of factors of production.

Any good index of productivity must pass several checks. The first requirement is the suitability to the purposes it is presumed to attend. By and large, the students of productivity are interested in the *competitiveness* of a firm, industry or economy. Some of them are also interested in the relationship between productivity and *social welfare* or *economic growth*. The second requirement of a good index of productivity is that movements in this variable should be related to technical change. Here we shall adopt a concept of technical progress so broad as to include changes in the methods of production, economies of scale, improvements in organization, learning by doing, etc. On the contrary, we should reject indexes that record changes in productivity when only distribution or output composition have changed.

In the light of the preceding criteria, we shall search in section 2 a disaggregated index of productivity suitable to measure sectoral competitiveness. Our conclusion will be that the best measure is the total labor embodied in any unit of final product. Or, to use a concept introduced by Pasinetti (1973) and rooted in Sraffa (1960), the labor employed in the *vertically integrated sectors* corresponding to the net output. In section

3 we shall discuss the proper way to get an aggregate index able to measure the potential social welfare and the potential growth. In section 4 we will review critically the usual indexes of productivity based either in "direct labor" or in "total factors". In the last section we will apply and compare the preceding indexes to the Spanish economy, during the years 1970-1988.

Some authors have already pointed out the shortcomings of the traditional measures and the advantages of those based on vertically integrated sectors (Rymes, 1972; Peterson, 1979; Wolff, 1985). Surprisingly enough, the first ones continue to be used everywhere. Maybe some clarifying remarks, like the ones we are going to make in this paper, were still necessary. And maybe it was not the right time to propose a change in the conventional paradigm. We hope that in the new era of cybernetics (where small computers are able to invert huge matrixes) and in the new political era (where Marxism - communism is no longer seen as a threat) , we expect that under these new circumstances students of all streams will raise less objections to the use of some tools of Classical Political Economy that have proved to be coherent and useful.

## **2. SECTORAL PRODUCTIVITY AND COMPETITIVENESS.**

### **2.1. VERTICALLY INTEGRATED SECTORS AND TOTAL LABOR PRODUCTIVITY.**

The conventional input-output analysis classifies the economy in "*industries*" that produce an homogenous output. In each industry is included (or should be included, if data were available) circulating capital, fixed capital and labor directly used in the process of production. The analysis in terms of *vertically integrated sectors* (from now on, VIS) offers a new arrangement of the same economy. It builds technically autarchic sub-systems, i.e., sections of industries able to produce all the inputs necessary to come up with the final output they produce. There will be a VIS for each of the goods composing the net final output. The labor (and capital) of each VIS is supposed to include not only people (and machines) directly employed in the production of final goods, but also those employed in the firms producing the inputs, and the inputs of the inputs.

To obtain the relevant variable of the VIS from the data of the actual sectors of the economy, one has just to multiply these data by the operator  $[I-A^+]^{-1}$ . This is Leontief's inverse matrix with the peculiarity that the matrix of technical coefficients ( $A^+$ ) contains, not only the consumption of circulating capital, but also the consumption of fixed capital. Both are technical requirements of production and we should account for them in order to obtain the autarchic units that the VIS are supposed to be. The vector of total labor coefficients ( $l'$ ) is obtained by multiplying the vector of direct labor coefficients ( $l$ ) by the above operator.

$$[1] \quad l' = l[I - A^+]^{-1}$$

In the same way, the matrix of fixed capital coefficients ( $k'$ ) corresponding to VIS derives from the capital/output coefficients of the actual sectors of the economy ( $k$ ).

$$[2] \quad k' = k[I - A^+]^{-1}$$

Pre-multiplying  $l'$  by the diagonal vector of final demand or net product we get the vector of the labor employed in each VIS ( $L' = l' \cdot \hat{y}$ ). These figures differ from the labor employed in the production of the output in the actual industries of the economy ( $L = l \cdot \hat{q}$ ). But, obviously, the summation of the labor employed in the VIS and the summation of labor employed in the actual sectors are equal to the total labor (scalar L). We can write:

$$[3] \quad L = l' \cdot y = l \cdot q$$

At this point, the computation of **integral productivity** or **total labor productivity** is immediate. We have just to divide the (net) output of each VIS ( $y_i$ ) by the labor it employs ( $L'_i$ ). This ratio is nothing but the inverse of the labor coefficient of the corresponding VIS.

$$[4] \quad p'_i = \frac{y_i}{L'_i} = \frac{1}{l'_i}$$

The preceding formula fulfills -we hope- all the requirements of a good index of productivity. To start with, it is simple. But the simplicity does not impair its generality

and coherence. Apparently only one factor of production (labor) is considered. Yet, the remaining factors (fixed and circulating capital) are included in it as “indirect labor”. Apparently, we deal with independent sectors. Yet they are “vertically integrated”, i.e. they take into account the interdependence of all industries.

Another quality of the preceding index is that variations in productivity are related to technical change; not to distribution or output composition. And the evaluation of these changes is clear. Even in the cases where the decrease in one factor (say, labor) comes with the increase in other factors (say, capital), we can appreciate whether they imply a forward or backward movement in productivity. We have just to see if total labor diminishes or augments.

A glance at the *production possibility frontier*, derived from VIS, will help us to understand the issue at stake. Let us consider an economy which produces only two final goods ( $y_1$  and  $y_2$ ). If all labor is employed in VIS 1, the maximum net product of this sector will be:  $L \cdot p_1'$ . If all labor goes to VIS 2 we get the maximum net product of 2:  $L \cdot p_2'$ . Assuming constant returns to scale, we can derive the production possibility frontier by drawing a straight line between both points. Or we can divide by L and draw the straight line afterwards. Such is the kind of frontier represented in *figure 1*. The vertical intercept stands for the productivity of VIS 1. The horizontal intercept measures the productivity in VIS 2. Any type of technical change that reduces total labor will cause an upwards shift of the frontier. In our example we have supposed that the change in productivity occurs only in industry 1, and so, the largest shift is registered in the vertical axis. Yet, VIS 2 captures some gains in productivity, since less labor is embodied in the inputs it takes from industry 1.

**Figure 1**

The error one makes when computing productivity from direct labor (i.e. labor employed in the actual sectors of the economy) may be significant. To figure it out, we have just to project the frontier to the right and to the left. B stands by the situation in which all labor is employed in sector 2, importing the necessary inputs of good 1. In points A and A' all labor is employed in sector 1, importing the requirements of good 2. According to the conventional measures, the increase in productivity in sector 1 would be measured by the segment A-A', while in sector 2 it would be considered nil. From our

Sraffian point of view, in sector 1 productivity increases  $(\mathbf{p}'_{1(1)} - \mathbf{p}'_{1(0)}) < (A' - A)$ .

Productivity in sector 2 increases  $(\mathbf{p}'_{2(1)} - \mathbf{p}'_{2(0)}) > 0$ .

## 2.2. PRODUCTIVITY AND COMPETITIVENESS.

The advantages we have encountered in the preceding measure of productivity would be irrelevant unless it was a good indicator of competitiveness. Although modern economists like to talk about "non price competition", prices continue to be the basic form of competition and the main indicator of competitiveness. Why bother searching other indicators if market prices are easily known? Here it is the first objection we have to answer.

The first reason is the interest to know the forces that make prices change. No doubt, technical progress is a major one, and it should be isolated. A second reason, is that prices observed in the market may differ from long run equilibrium ones. At most, market prices reflect a short run equilibrium, i.e. prices that "clear the markets", adjusting supply and demand. Long run equilibrium requires, in addition, the uniformity of the rate of profit earned by the representative firms of each industry. This condition is fulfilled by prices of production, that are supposed to work as the "gravity centers" of market prices.

When we operate in terms of VIS, the prices of production can be broken out in wages and profits:

$$[5] \quad p = wl' + rp k'$$

Taking  $w$  as *numeraire* and dividing by  $l'$ , we get the following relationship for any good  $i$ :

$$[6] \quad \frac{p_i}{l'_i} = 1 + \frac{rpk'_i}{l'_i}$$

The preceding expressions show that there are three wedges that may cause a deviation of prices of production ( $\mathbf{p}$ ) from labor values (here identified with  $\mathbf{l}'$ ). Such wedges are the real wage ( $\mathbf{w}$ ), the rate of profit ( $\mathbf{r}$ ) and the capital / labor ratio measured in terms of vertically integrated sectors ( $\mathbf{pk}'/\mathbf{l}'$ ).

First we shall look to the real wage, or better, to the *unit labor cost* defined as:

$$[7] \quad ulc' = wl_i' = \frac{w}{p_i}$$

Clearly, if the increases in productivity in one sector are entirely absorbed by the wages of the same sector, prices of production would not reflect changes in productivity. But, as many authors have evidenced (for instance, Baumol, 1967), wages usually increase *pari pasu* in all sectors, regardless of the sectoral evolution of productivity. In the international context, the wage gap may be a huge one, but not an ever growing one<sup>1</sup>.

To be a proper index of productivity, the unit labor cost should be referred to VIS, as it is shown by the dash ( ' ) on  $l$  and  $p$ . This is not the normal procedure. In the conventional literature the unit labor cost is defined by the ratio between the wage and the "value added per worker" in the actual sectors of the economy. From this definition they derive a straightforward conclusion: to maintain the international competitiveness, firms cannot endure increases in wages higher than the increases in productivity. This conclusion is not necessarily true. Even if the technical conditions of a firm do not change, it can get a part of the productivity gained in other industries and it can allow increases in wages without damaging the rate of profit or its international competitiveness.

The other two wedges between labor-values and prices of production are the rate of profit ( $r$ ) and the capital - labor coefficients ( $pk'/l'$ ). Both wedges do not cause any significant deviation whenever they remain stable through time. Notice, in addition, that the relevant coefficients do not refer to the actual sectors of the economy ( $k$ ), but to the vertically integrated ones ( $k'$ ). Those are supposed to be more uniform, since they are a kind of weighted mean, obtained by multiplying  $k$  by  $[I - A^+]^{-1}$  (Pasinetti, 1973, and Parys, 1982).

“Prices of production (and market prices) necessarily deviate from labor values”. This is all we can say from a theoretical point of view. To figure out the scope of such

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<sup>1</sup> Krugman (1994, appendix to chapter 10) claims that productivity and competitiveness are unrelated. This conclusion rests on the hypothesis that wage movements offset productivity changes. What we have said above, questions Krugman's hypothesis and conclusions. A country whose industries are left behind in the productivity race, will certainly lose many jobs.



deviation we had better look at the empirical evidence. David Ricardo (1823), committed to the defense of the labor theory of value, guessed that it could explain up to 93% of the level and movements of relative prices. Current evidence for the American economy (Ochoa 1984, 1986 and 1989; Shaikh, 1995) validate Ricardo's intuition. Other key references are Petrovic (1987) for the Yugoslav economy, and Cockshott, Cottrell & Michaelson (1995) for Great Britain. In section 5 we are going to see how good the fitness is for the Spanish economy.

### 3. AGGREGATE INDEXES OF PRODUCTIVITY AND THEIR ABILITY TO MEASURE SOCIAL WELFARE AND POTENTIAL GROWTH.

#### 3.1. TOTAL LABOR PRODUCTIVITY AT THE AGGREGATE LEVEL AND SOCIAL WELFARE.

So far our main concern has been "competitiveness" and we have looked at the industry level, since it is clear that competition occurs among industries and firms, not among nations. This does not imply that aggregate indexes of productivity are useless. They can provide useful indicators of social welfare and potential growth. In any case, the index should be coherent and people using it should be aware of its weaknesses and strengths.

Assuming that the indexes of sectoral productivity derived in section 2.1 are correct, we can obtain an aggregate index by a weighted mean of them. The proper weight is the share of total labor employed in each VIS ( $L_i' / L$ ). Applying this criterion and having in mind that  $L_i' = l_i' \cdot y_i$ ;  $p_i' = 1/l_i'$ , we obtain the following result, where  $y_i$  stands for the net output of industry  $i$ :

$$[8] \quad \Pi = \sum_i p_i' \cdot \frac{L_i'}{L} = \sum_i p_i' \cdot l_i' \cdot \frac{y_i}{L} = \sum_i \frac{y_i}{L} = \frac{Y}{L}$$

Let us illustrate these relationships with **figure 2**. It represents the *technological frontier of distribution and growth*. For any technology, an inverse relationship exist between the real wage ( $\mathbf{w}$ ) and the rate of profit ( $\mathbf{r}$ ), on the one hand; and between consumption per worker ( $\mathbf{c}$ ) and the rate of growth ( $\mathbf{g}$ ), on the other. This relationship

can be linearized if we choose the right numeraire. In addition we will obtain the equalities ( $w=c$ ;  $r=g$ ) if, for simplicity's sake, we assume the classical expenditure hypothesis: all wages are consumed and all profits are saved. In this context, any increase in total labor productivity will be reflected in an upwards shift of the technological frontier (the intercept with the vertical axis will be higher). This intercept reflects the (potential) **consumption per worker** that we take as a proxy of the (potential) **social welfare**.

**Figure 2**

The main problem with aggregate measures of productivity is that they are altered by changes in the composition of output, completely alien to technical change. In the last formula we can see that the evolution of  $\Pi$  depends on both changes in technology affecting sectoral productivity ( $p_i'$ ) and changes in the composition of net output, which is reflected in the share of workers employed in each VIS ( $L_i' / L$ ). To understand the relevance of this problem, we have just to imagine two countries that in year  $t$  produce the same goods with the same technologies.  $n$  years later, these technologies continue in use, but country 1 has specialized in (labor intensive) services, and country 2 in (capital intensive) manufactures. The ratio  $Y/L$  is bound to diminish in country 1 and to augment in country 2. Yet, no real change in productivity has occurred. The competitiveness of national industries remains constant; and so happens with social welfare.

If we desire that the ratio  $c=Y/L$  be a good connector between technology and social welfare, i.e. an index independent of output composition, we should redefine conveniently the vector of net product. Our proposal is that it incorporates only consumption goods in the same proportions that it appears in the actual economy (as reflected in family budget statistics). In comparing two years, looking for the gains in social welfare, we should examine the following expression:

$$[9] \quad \Pi = \sum_i p_i' \left( \frac{l_i' y_i^*}{L} \right) = \frac{Y^*}{L}$$

$y_i^*$  stands for the net product of consumption good  $i$ , when Net National Product appears in the proportions suggested in the preceding paragraph

Notice that the aggregate measure of productivity we have proposed coincides with the conventional measure “value added per worker”. Both measures are different at the sectoral level but coincide for the aggregate economy. This coincidence may seem paradoxical. In the derivation of sectoral productivity we have criticized the conventional measures that result from dividing the total product or the value added of an industry by the direct labor employed in it. Now, for the national economy, we are accepting a formula that coincides with the conventional measure “value added per worker”. To solve this paradox, we have to recall that Macroeconomics conceives the national economy as a huge vertically integrated sector producing a basket of “final” goods called “net national product”. This basket is represented in the numerator, while the denominator refers to total labor, i.e. labor employed in the production of final goods, plus labor employed in the production of intermediate goods and capital goods for replacement.

### **3.2. TOTAL CAPITAL PRODUCTIVITY AND THE MAXIMUM RATE OF GROWTH.**

In the preceding section we have seen that the intercept of the technological frontier with the vertical axis stands as an index of the (potential) social welfare. In this section we shall see that the intercept of the same frontier with the horizontal axis is a good measure of the (potential) growth of the economy.

The first step in deriving this index is the computation of the matrix of capital/output coefficients in vertically integrated form (see matrix  $\mathbf{k}'$  in formula [2] ). The maximum eigenvalue ( $I$ ) of this matrix is the inverse of the index we are looking for. We can call it **integral productivity of capital** or **total productivity of capital** ( $g$ ).

$$[10] \quad g' = \frac{1}{I}$$

This ratio is a pure number that depends exclusively on technical factors and it is immune to changes in distribution or output composition. This advantage is so convenient that some Sraffians consider it the best index of productivity (Sánchez Choliz, 1990). Yet, we should be aware of what the index can measure and what the limits are.

The inverse of the maximum eigenvalue of matrix of the matrix of capital coefficients is Sraffa's standard ratio (R) and indicates the maximum rate of growth and the maximum rate of profit in an economy where the entire surplus is appropriated by capitalists, who save and invest it. The necessary consumption of workers (per unit of output) is supposed to be included in matrix  $A^+$ , together with the consumption of intermediate goods and capital goods for replacement. In order to have a useful measure of productivity and growth, we should consider as necessary consumption the whole consumption of workers. But to maintain the independence from distribution and expenditure patterns, we should fix this "necessary consumption basket". What we propose, it to take the actual consumption basket of year t and assume that it remains constant during all the period analyzed. Under these assumptions, the intercept of the technological frontier with the horizontal axis will stand for the maximum rate of growth corresponding to a given technology and given expenditure patterns.

Notice, however, that  $g$  says nothing about the objectives to which productivity measures are usually referred: competitiveness and social welfare. Even worse, a new technique may allow increases in total labor productivity (a proxy of the potential social welfare) and decreases in total capital productivity (a proxy of the potential growth). This is the case reflected in figure 2 and corresponds to a usual form of technical change that brings about labor savings by introducing more mechanized methods of production.

## **4. A CRITICAL REVIEW OF THE TRADITIONAL MEASURES OF PRODUCTIVITY.**

### **4.1. DIRECT AND APPARENT PRODUCTIVITY OF LABOR.**

The most elemental index of the sectoral productivity stems from the ratio of total output of any industry ( $q_i$ ) to the labor directly employed in it ( $L_i$ ). This ratio can be called *direct productivity of labor* and coincides with the inverse of the sectoral labor coefficients:

$$[11] \quad p(d)_i = \frac{q_i}{L_i} = \frac{1}{l_i}$$

The simplicity (both at the theoretical and empirical level) explains its huge diffusion in applied studies. Yet the errors it can cause are obvious. We shall illustrate them with an historical example. During a time European politicians were proud of the great advances in agricultural productivity: half the peasants were able to produce twice as many goods. Yet, at the crucial moment it was shown that the difference in costs with American products had increased. Politicians had been too naive; they forgot to compute the increase in intermediate goods and fixed capital used by agriculture. As we have seen in figure 1, direct productivity overestimates the advances in productivity in the sector experiencing technical change and ignores the diffusion of productivity to the remaining sectors.

To avoid this problem national statisticians have introduced another index: **value added per worker**:

$$[12] \quad p(a)_i = \frac{VA_i}{L_i}$$

By this procedure, overestimation of productivity may disappear, but the roots of the problem are not removed. Instead of augmenting the denominator to include direct and indirect labor, they reduce the numerator in an unjustified way. The ratio  $q_i/L_i$  had, at least, a clear economic meaning. It pointed out to the industry introducing technical change. It could be computed both from a deflated input-output table or from one expressed in physical units. The ratio  $VA_i/L_i$  lacks a precise economic meaning. It can be computed only from a table expressed in prices; and to do so, it is necessary to introduce the double deflation method, whose validity has been questioned in the literature (Rymes, 1972; Dalgaard, 1990)<sup>2</sup>.

Summing up. The traditional measures of sectoral productivity are not good indicators of competitiveness. They consider only one factor of production (labor) in one isolated sector. They miss the interrelationship of factors and sectors, and so, they cannot account properly for costs and prices.

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<sup>2</sup> The problem affects mainly profits. Why and how to deflate profits? The practical procedure is to define "real value added" as the difference between deflated total output and deflated intermediate consumption.

## 4.2. "TOTAL FACTOR PRODUCTIVITY" IN NEOCLASSICAL ANALYSIS.

Being aware of the shortcomings of the simple measures of productivity, academic economists have introduced the idea of *total factor productivity*. Solow's (1956) pioneer study was at the aggregate level. Afterwards the methodology was applied to analyze sectoral productivity. Nowadays studies have become more sophisticated by introducing new elements such as the degree of capacity utilization, economies of scale, the qualifications of workers by means of investment in human capital, and so on. No doubt, such elements have improved the neoclassical measure of productivity. But is it well grounded? Let us recall the roots of the model and figure out what they purport.

$$Y = e^{\lambda t} \cdot K^{\alpha} \cdot L^{\beta}$$

[13]  $\dot{Y} = \lambda Y + \alpha \dot{K} + \beta \dot{L}$

$$I = \dot{Y} - \alpha \dot{K} - \beta \dot{L}$$

The first equation in [13] describes the economy by means of a Cobb-Douglas production function. Combining capital and labor the economy produces a level of income  $Y$ .  $\alpha$  is the elasticity of the net product to the capital factor; in equilibrium it coincides with the marginal product of capital and with the share of profits in net income.  $\beta$  is the elasticity of the net product to the labor factor; in equilibrium it coincides with the marginal product of labor and with the share of wages in national income. (Whenever the production function exhibits constant returns to scale -as is the usual hypothesis-  $\alpha + \beta = 1$ ). Lastly,  $e^{\lambda t}$  stands for total productivity of factors. This index is supposed to grow through time ( $t$ ) at a constant rate  $\lambda$ . Taking time derivatives (as shown in the second and third equations), we can define *total factor productivity growth* as the difference between the actual increase in production and the part that can be explained by the accumulation of capital and labor.

What supports the preceding equations is the neoclassical production function. The problems of this function are well known but ignored. To start with, we should be aware that the very concept of "marginal productivity" requires a malleable production function where labor and capital can be mixed at will. This kind of representation will only be found in Economics textbooks; entrepreneurs and engineers would never rely on

it. The specific form assumed for the production function is also unfounded. Economists have used a Cobb-Douglas because the estimates of marginal productivity are akin to those predicted by neoclassical economics and those registered in real life. Yet, Shaikh (1974) showed that this coincidence will necessarily occur, provided the share in income of profits and wages was constant enough, as happened after War World II. By no means it proves that the economy works as a Cobb-Douglas production function.

From a theoretical point of view we cannot omit a reference to the capital debates of the sixties, where Cambridge-England beat Cambridge-Massachusetts<sup>3</sup>. The problem stems from the two-side nature of capital. On the one hand it is a factor of production, whose demand price is related to productivity; on the other hand, it is a produced good, whose supply price depends on the cost of producing it. Technical progress allows to produce goods in a more efficient way. Usually it comes about by the introduction of new capital goods that are also more efficiently produced and, therefore, cheaper. The neoclassical procedure, that values capital goods at their initial price, is unjustified<sup>4</sup>.

When dealing with aggregate productivity the problems are different. In section 3.1 it was shown that the ratio “net product / total labor”, is a useful index for social welfare comparisons. We emphasized that in the denominator should be included the workers producing the final goods which define the “national net product”, but also the workers producing circulating and fixed capital for replacement. To add the capital factor -as neoclassical studies inspired by Solow purport- would be *redundant* and *misleading*. It stands as an example where too much effort, pushes us away from the goal.

#### **4.3. "TOTAL FACTOR PRODUCTIVITY" IN INPUT-OUTPUT ANALYSIS.**

Following Kendrick (1961), several authors have analyzed total factor productivity at the industry level with the information provided in input-output tables. If we divide the figures of each column by the total output of the industry we obtain the

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<sup>3</sup> For a general review of the debate see Harcourt (1973). The implications for productivity indexes are analyzed by Rymes (1972).

<sup>4</sup> Notice, also, that "total factor productivity" accounts for all factors of production, but it continues missing the interdependence of all industries.

coefficients  $\mathbf{b}_{ij}$ . They stand for the typical technical coefficients of intermediate goods, but also for labor and capital payments per unit of output. Let us suppose that after a process of technical change there is a decrease in the requirements of some inputs of industry  $j$ . If all intermediate and primary inputs are valued at the current prices the sum of  $\mathbf{b}_{ij}$  elements will be one, by definition. But, whenever inputs are valued at their prices in the base year, then  $\sum_i b_{ij} < 1$ , and its inverse -which stands as an index of productivity- will be higher than one. The increase in productivity in sector  $j$  will be measured by the formula:

$$[14] \Delta p(t)_j = \frac{1}{\sum_i (b_{ij})^o} - 1$$

By this procedure, output is referred to all factors that cooperate in the process of production, avoiding the problem of overvaluation that we encountered in the simpler indexes of sectoral productivity. Yet, the formula continues to miss the interdependence among all industries and, therefore, is unable to capture the diffusion of productivity from the most dynamic sectors to the remaining ones.

In any case, Kendrick's analysis may be a good starting point for further refinements that do take into account the interdependence of production. Courbis-Templé (1975) and Fontela (1989, 1993), compare the input-output table of year  $t$  at current prices with the same table at the prices of the base year. The difference between the actual and the deflated table is supposed to show the gains in productivity in the innovating sector and the diffusion to the remaining ones. Whenever factor incomes remains constant, technical change in sector  $j$  will show up in a price fall of good  $j$ , a fall that will reduce costs and prices of all industries purchasing such a good<sup>5</sup>.

By this procedure, the role of prices is emphasized as the transmission mechanism of technical change. This idea is akin to our conclusions in **section 2.2**. We shall just recall the warnings there indicated. (1) The prices that accomplish this role are prices of production. Market prices tend towards them, but at the moment of observation they could be far enough. It is risky to draw conclusions on the basis of prices that are changing. (2) Not all the changes in prices are due to technical progress. If we want to



analyze the true impact of technical change, we had better focus on the quantity system embedded in input-output tables.

## 5. PRODUCTIVITY INDEXES APPLIED TO THE SPANISH ECONOMY (1970-1988)

In this section we are going to compare the evolution of productivity in Spain according to different indexes. Barriga (1992) provides coherent data for the period 1970-1988 aggregated in nine sectors. Notwithstanding the reader should be aware of the following limitations in the data:

(1) The prices of reference should be “production prices”. Yet, only “market prices” can be observed. We are bound to suppose that, as a mean, market prices are close to the long run equilibrium prices of production<sup>6</sup>.

(2) The relevant category of capital to compute the rate of profit and prices of production is “fixed capital”. We should know the stock of capital per unit of output for all sectors and all capital goods. Yet the only matrix we can get refers to the flows of circulating capital and depreciation per unit of output. We are bound to suppose that both matrixes move *pari pasu*.

The regression will be run on the following equations:

$$\begin{aligned}
 [15] \quad \log(p_{i,t}) &= \mathbf{a}_0 + \mathbf{a}_1 \log(l'_{i,t}) + \mathbf{q}_{i,t} \\
 \log(p_{i,t}) &= \mathbf{b}_0 + \mathbf{b}_1 \log(l_{i,t}) + \mathbf{e}_{i,t}
 \end{aligned}$$

In the first equation we test the relationship between prices and total labor (i.e. the inverse of total labor productivity).  $\mathbf{q}_t$  accounts for the factors that may cause deviations (namely, the organic composition of capital and the rate of profit). Our hypothesis is that these factors are not relevant and  $\mathbf{q}_t$  will be a white noise. In the second equation we test the relationship between prices and direct labor (i.e. the inverse of direct labor productivity).

<b>Table 1</b>
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<sup>5</sup> The deflation of value added requires special care: we have to know the number of people employed and the stock of capital in real terms. The double deflation method is considered illegitimate by these

The results of the regressions are summarized in table 1. Clearly the estimation of prices from total labor is quite good for all sectors except one <sup>7</sup>. And it is better, than the estimation of prices from direct labor, although there are three sector where the differences are small <sup>8</sup>.

Our hypothesis that the organic composition of capital does not cause serious deviations between labor values and prices is validated. In **figure 3** we can see that the difference in the organic composition of capital among agriculture and manufactures is not a big one and remains constant through time (as a matter of fact the divergence diminishes).

**Figures 3, 4 and 5**

In **figure 4** we observe the evolution of total labor productivity in the same sectors. The larger increases in productivity occur in the production of intermediate goods. In the remaining industries the rise in productivity is lower and more similar. **Figures 5** and **6** show how well the standard measures of productivity track the evolution of total labor productivity that, as we have seen, moves quite close to prices. In agriculture, direct and total labor productivity move together, while value added per worker seems to be out of touch. The opposite occurs, in the intermediate goods industry. Direct labor productivity rises far above total labor productivity and value added per worker. We do not have room to reproduce the evolution of productivity in all sectors, but we can say that in general the evolution of value added per worker is less reliable as a measure of sectoral productivity.

Let us turn to the aggregated indexes of productivity. **Figure 7** shows the evolution of aggregated productivity, measured as “consumption baskets per worker”. It has been computed as the inverse of the total labor required to produce an homogenous basket of consumption goods (the basket that people used to consume in

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authors.

<sup>6</sup> To standardize, all prices have been divided by the nominal wage.

<sup>7</sup> Sector 8 stands for “Marketed services” (other than “Transport and communication”). Since it is a sector dominated by small firms, data on physical units of production and deflators are not reliable.

<sup>8</sup> We can use some Euclidian measures for the deviations (in the aggregate) of  $\alpha$  and  $\beta$  from the unit vector (which is the optimal result). (Sector 8 has been omitted). The results are:

(a) Mean Absolute Deviation:  $1/n \sum |1-\alpha_i|$ : 0.166 for  $\alpha$ ; 0.301 for  $\beta$ .

(b) Mean Squared Deviation:  $1/n [\sum (1-\alpha_i^2)]^{1/2}$ : 0.063 for  $\alpha$ ; 0.123 for  $\beta$ .

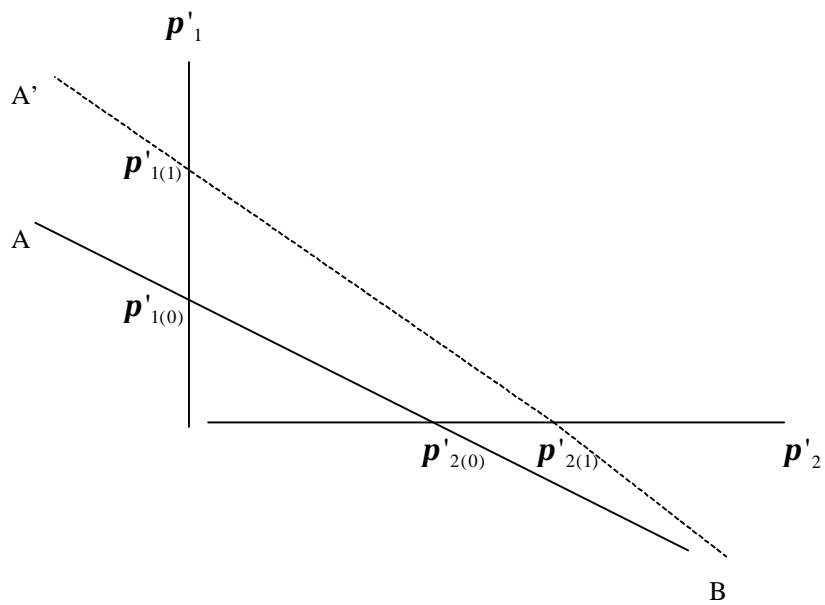
1980). The increase of social welfare was remarkable at the beginning and the end of the period analyzed. But it was nil during the years 1975-83.

**Figure 8** shows the evolution of the maximum profit margin, that is defined by the ratio of profits to total current costs. It coincides with the inverse of the maximum eigenvalue of the matrix of capital flows (intermediate goods, capital goods for replacement and consumption goods per workers). The basket of consumption goods has the composition of 1980, while the level of consumption coincides with total wages in 1970. If the flows of capital here considered move *pari pasu* with the stock of fixed capital, the index stands as a measure of the maximum rate of profit and the maximum rate of growth.

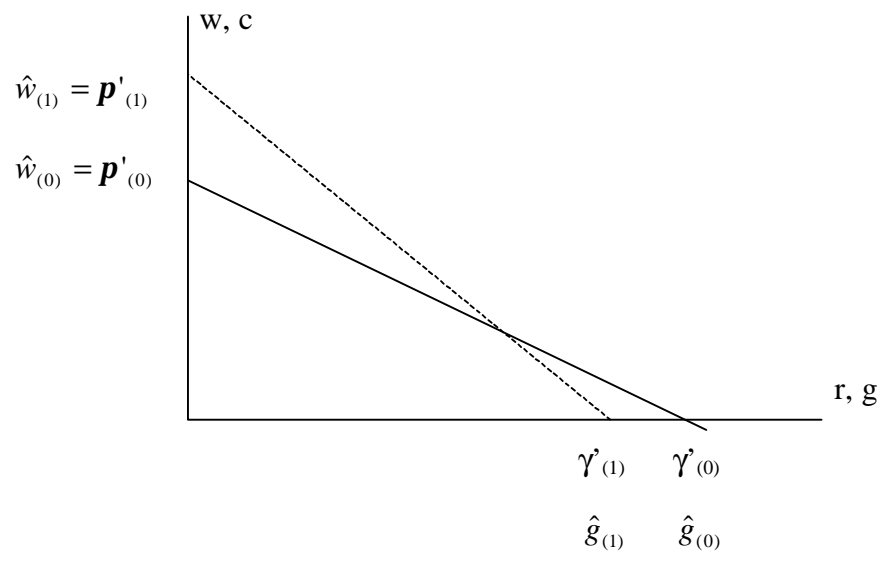
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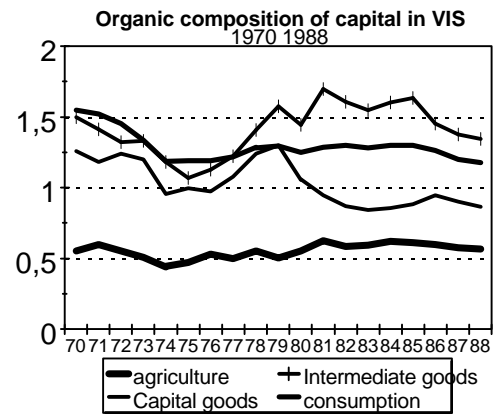
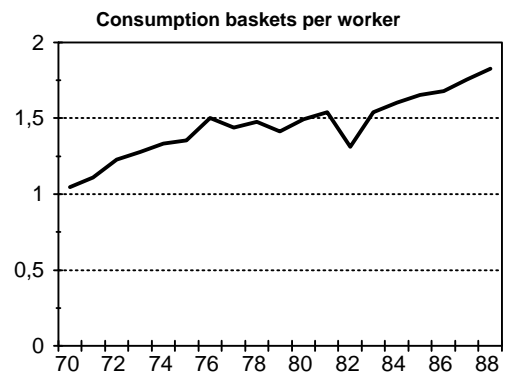
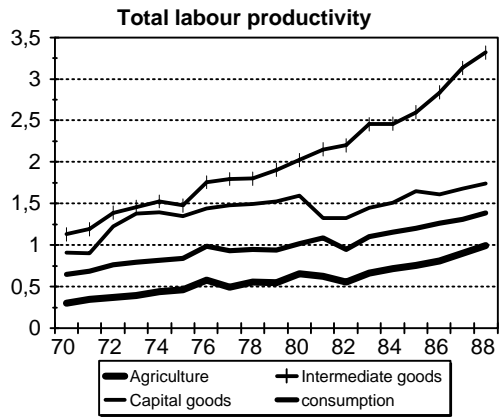
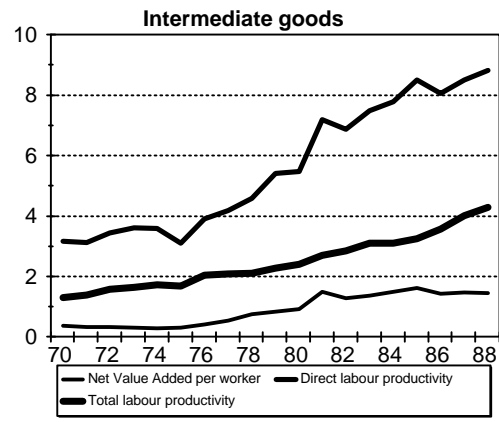
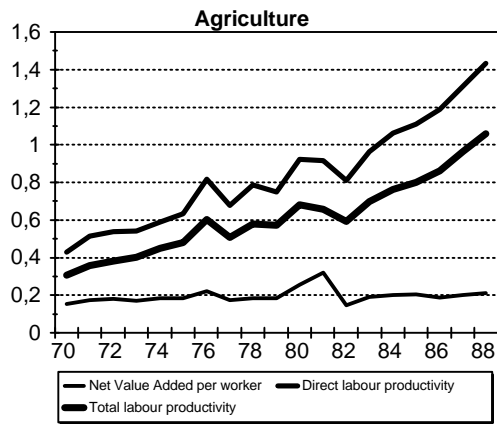
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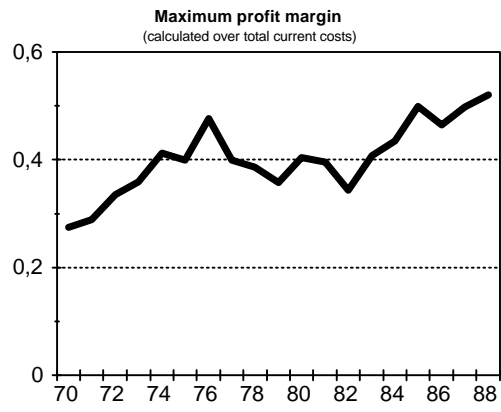


**Figure 1: Production possibility frontier and total labor productivity**



**Figure 2: Frontier of distribution and growth. Total productivity of labor and capital.**









**Table 1: Explanation of prices by total labor ( $\alpha$ ) and direct labor ( $\beta$ )**

Main results from least ordinary squares in [15] referred to the Spanish Economy 1970-1988.

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Sector	$\alpha_1$	$\beta_1$	$R^2_\alpha$	$R^2_\beta$	t-stat $\alpha$	t-stat $\beta$
1 Agriculture	1.004	0.972	0.93	0.93	15.17	14.93
2 Energy	0.672	0.370	0.79	0.62	7.95	5.28
3 Intermediate goods	0.905	0.671	0.87	0.76	10.88	7.39
4 Capital goods	1.209	0.778	0.79	0.28	8.06	2.63
5 Consumption	1.113	1.276	0.88	0.73	11.43	6.81
6 Building & Construction	0.744	0.562	0.62	0.32	5.27	2.87
7 Transports & Communications	1.036	0.889	0.89	0.92	12.03	14.09
8 Marketed Services	0.124	-0.273	0.01	0.01	0.33	-0.49
9 Non marketed services	0.716	0.626	0.67	0.61	5.89	5.16