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Projecting the Development of Russian Economy Using the Dynamic Input – Output Model with Fuzzy Parameters ¹

Estimates for projecting the development of the Russian economy for the period 2008-2012 were carried out in two stages.

1. A forecast was carried out with the help of a deterministic Dynamic Input-Output Model.

2. Taking into account the results of projecting the development of the Russian economy estimates based on the Dynamic Input - Output Model with fuzzy parameters were carried out.

1. Hypotheses underlying various estimates based on the Dynamic Input - Output Model for the years 2008-2012

The forecasting estimates for the years 2008-2012 were carried out with the help of the Dynamic Input - Output Model based on the information data of 2007. At the same time, all the estimated parameters were defined in the comparative prices of 2003.

The main goal of the forecasting estimates was to investigate whether it would be possible for Russia to reach the level of per capita GDP output close to that of the least developed countries of Western Europe - Greece and Portugal in the course of the next decade (2008-2018). In 2002, taking into account the parity of the purchasing power of national currency, GDP per capita in Russia was approximately two times lower than in these countries [1, p. 769, 771]. As in the course of ten years GDP at least doubles, its average annual growth rate should account for not less than 7.2%. In the period 2008 - 2012, due to this average annual growth rate, GDP should grow at least 41%.

The possibilities of attracting additional workforce into the production process being limited, the main source of production growth in Russia is raising its efficiency. Indeed, in the period of economic recovery (1999-2006), employment in the Russian economy grew by 7.9%, while the productivity of labour grew by 53.7% (see table 1). The growth of productivity was the crucial factor that provided the increase of GDP by about two-thirds in the given period.

The main source of productivity growth is simple and extended reproduction of fixed assets carried out through investments. Investments into fixed assets that provide the replacement of morally and physically outdated active part of fixed assets provide the "entrance" of new technologies into the production process. The comparison of the fixed capital investments growth rate and the rates of labour productivity growth in 1999-2006 (see table 1) shows that 1% of growth of the first index accounted for 0.46% of growth of the second one. (53,7/117,5=0,46).

As we can see, the most important condition for achieving the goal of increasing GDP per capita about two times in 10 years when labour resources are limited is to considerably increase fixed capital investments. If we proceed from the above ratio of the correlation between the increase of the rate of investments into fixed capital and the growth of productivity and suppose that the size of labour resources in the Russian economy does not grow, it will be possible to forecast that in five years the essential fixed

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capital investments growth rate will account for not less than 90% (41%/0.46 90%) or 13.7% per year on average.

Another justification for high rates of investment into fixed capital in the projected period is connected with the need for substantial renewal of basic assets that should be carried out in the next few years.

The need for such research is well grounded since the moral and physical depreciation of fixed assets in Russia during the last decades has reached such a degree that only a rapid growth of production of asset-building industries is able to provide considerable production growth and raise the living standard of the population. This paper continues the research in this field carried out earlier. [4].

Index	Growth rates in 1999 - 2006
Gross Domestic Product	165,8
Expenditures for Final Consumption	170,0
Employment in the Economy	107,9
Fixed Assets	105,5
Labour Productivity with respect to GDP	153,7
Capital Productivity	157,1
Fixed Capital Investments	217,5
Fixed Assets Put in Service	211,5
Average annual growth of GDP per 1% of investments into	
fixed assets, %	0,56
Average annual growth of labour productivity per 1% of fixed	
capital investments, %	0,46
Average annual growth of final consumption per 1% of fixed	
capital investments, %	0,6

Table 1. Major macroeconomic indices growth rates in the economy of Russia in 1999 - 2006, %.

References : [1], [2], [3].

In the period 1999 - 2006, fixed capital investments growth rates accounted for 218%, while the rates of putting fixed assets into operation was 212% (see table 1). However, in spite of the considerable growth of investments into fixed assets and the input of fixed assets, the absolute value of these indices in the comparative prices of 2006 remained equal to about 50% of the level of 1991. As a result, the Russian economy did not experience any important positive changes in the age composition of fixed assets and the degree of their depreciation.

In general, the degree of fixed assets depreciation in the national economy in 1998 was equal to 40.1%, in 2005 - 44.3% (see table 2), and in 2006 - 45.3%. The degree of wear and tear of machinery and equipment in 2006 was 52.5%. The degree of fixed assets depreciation in industry in 1998 accounted for 53.3% and in 2005 it was 49.7%. Due to an increased input of fixed assets in recent years, the ratio of fixed assets renewal in the economy in general, according to the estimates of the Federal Statistics Service of the Russian Federation, grew from 1.1% in 1998 to 2.2% in 2005; in industry it grew from 0.9% to 2.6% correspondingly (|see table 2). At the same time, the ratio of fixed assets retirement in the Russian economy in general remained the same (1.1%), while in industry it even fell from 1.3% in 1998 to 1.0% in 2005.

Note that in the USA that has an enormous production mechanism the ratio of fixed assets renewal in the end of the 90s (1998 – 1999) was considerably higher than in Russia and accounted for 5.2% (estimated on the basis of data [6, p. 496 – 497]).

The information given above testifies that in the years of the economic recovery *there were no noticeable positive qualitative changes in the condition of the fixed assets*. A high depreciation level of fixed assets is still one of the main reasons for unstable economic development and uneven living standard growth of the population in the medium and long-term periods.

The size of investments made in the recent years cannot lead to radical shifts in the age composition of production mechanism. The share of state in the pattern of fixed capital investments capitalization is still insignificant. In the last seven years (1998 -2006) the share of consolidated budget in fixed capital investments capitalization accounted for approximately 20% (see e.g., [10, h. 96]). If we take taxation, in 2002 there were abolished profit tax privileges for enterprises investing their financial resources into renovating and expanding fixed assets. Consequently, in the years of the economic recovery, no active fiscal policy was conducted that could contribute to a rapid renewal of fixed assets.

It seems that in the situation of a crisis condition the fixed assets the state should conduct a more active economic policy that would stimulate a rapid renewal of fixed assets. In the area of monetary policy there should be introduced a package of measures aimed at lowering the real interest rate for crediting business. In fiscal policy, investments should be stimulated through tax incentives as it was done before. However, in our opinion, the application of only indirect regulation instruments in the investment process seems insufficient in the present situation. In the condition of a balanced and profitable budget, the state represented by the Federal Center and the administrative bodies of the subjects of the Russian Federation may directly finance priority innovational investment projects in the sphere of creating infrastructure (construction of roads, airports and seaports, etc.) This variant of the economic policy in the area of investments would promote the transition of the economy to the path of innovational development that is stipulated for in one of the versions of the Russian Federation [11]. State financing should be carried out mainly on a competitive basis. The mechanism of decision-making should be astransparent as possible and be controlled by the society.

In this connection, it seems appropriate to research further the possibilities of a greater direct financing by the state represented by consolidated budget of some part of investments for renewing fixed assets.

A considerable growth of investments into fixed capital, especially into its active part, is also possible in terms of a fuller utilization of production capacities. The level of utilizing production capacities in machine building in Russia remains low. In 2005 production capacities in machine building fluctuated from 3.9% (production of bridge and electric cranes) to 68% (production of automobiles) (see table 3).

According to our estimates, on average, the degree of utilizing production capacities in industry in 2002 accounted for about 42% (the estimates are based on the data of Federal State Statistics Service of the Russian Federation). It opens new possibilities for improving considerably the production of the means of labour partly due to an increased utilization of production capacities in industry. However, the most important question concerning the possibility of actual utilization of these production capacities and producing competitive products on their basis remains open. Another opportunity for a large-scale renewal of fixed assets is to import large amounts of machines and equipment that would have a negative effect on the value of current account operations and balance of payments in general.

Due to a rapid growth of investments into fixed assets, their retirement compensation rate should grow considerably. According to our estimates, in order to provide a stable economic growth along the path of innovational development fixed assets retirement compensation rate should at least triple. For this reason, the estimates were based on the hypothesis that in 2012 this macroeconomic parameter would amount to approximately 3.3%. At the same time, the retirement compensation rate for the active part of fixed assets (machines and equipment) should grow from 1.6% in 2007 to 4.8% in 2012, while for the passive part (structure) it should increase from 1% to 1.5% accordingly. In addition, the value of fixed assets was supposed to grow at approximately the same rate as in the recent years. In other words, investments into fixed assets growth would equal 1%. Under the specified conditions of rising retirement compensation ratio, the growth rates of investments into fixed assets was determined endogenously in the course of solving the problem using the Dynamic Input-Output Model for the years 2008-2012.

Indices	1998	1999	2000	2001	2002	2003	2004	2005	Percentage changes in 2005 compared to 1998
Fixed assets depreciation rate in the country's economy as a whole	42,2	41,9	42,4	45,8	47,9	49,5	42,8	44,3	2,1
Fixed assets renewal ratio in the country's economy as a whole	1,1	1,2	1,4	1,5	1,6	1,9	2,1	2,2	1,1
Fixed assets retirement ratio in the country's economy as a whole	1,1	0,9	1,0	1,0	1,0	1,1	1,1	1,1	0
Fixed assets renewal ratio in industry	0,9	1	1,3	1,5	1,5	1,7	1,8	1,9	1
Fixed assets retirement ratio in industry	1,3	1	1,2	1,1	1	1,0	1,0	1,0	-0,3
Fixed assets depreciation rate in industry	52,9	55,1	51,6	52,3	51	52,9	51,4	49,7	-3,2
Share of equipment less than 10 years old in industry	24,2	19,3	15,3	13,3	12,5	12,7	No data	No data	-11,50 ¹⁾

Table 2. Dynamics of indices characterizing the state of fixed assets in Russia in 1998 – 2005, %.

1) Difference in data for 2003 and 1998

References: [1, p. 327-329], [6, p. 318].

The third aspect reflected in developing different variants of forecasting estimates with the help of the Dynamic Input-Output Model with fuzzy parameters implied the necessity to increase end consumption by the population not less than twice during the projected period. In 1999 – 2006, 1% of GDP growth accounted for 0.94% growth of end consumption (see table 1). On the basis of this ratio, GDP growth rate in ten years should account for about 206% (100 % + (100%/0,94)). During the forecasting period (2008 - 2012) GDP growth rate should reach 43%. In 1999 - 2006 the ratio characterizing the correlation between the growth rate of GDP and growth rate of investments into fixed assets was equal to 0.56% (see table 1). Consequently, in 2008 – 2012, investments into fixed assets should grow not less than 78% (43,5 % / 0,56 \approx 78%).

The above estimates following from the necessity to provide the required growth of end consumption and labour productivity make it possible to draw a conclusion that *the attainment* of target indices in the development of the Russian economy in 2008 - 2012 will require to increase investments into fixed assets by at least 80% - 90%.

The present parameter was adjusted in the course of forecasting estimates with the help of the Dynamic Input-Output Model.

The following assumptions are made in all the estimated alternatives.

1. It is assumed that by 2012 the size of net export of the Russian economy will fall. Under the optimistic scenario it is expected to fall by 33% and under the pessimistic one by 8%.

A greater decrease of balance in the optimistic scenario is justified by the fact that according to this scenario there will be a much higher rate of GDP growth and, consequently, a much higher rate of import growth. On the whole, the specified net export dynamics is much more optimistic in comparison with the scenario of the development of the economy of the Russian Federation prepared by the Ministry of Economic Development and Trade of the Russian Federation. According to the projection made by the ministry, already by 2010 net export will approach zero value [11]. Such development of events seems too pessimistic.

2. The size of labour resources and the population of Russia remain constant during the whole forecasting period and are equal to the values of these indices in the base year. In other words, it is assumed that the measures taken by the Government of the Russian Federation to stimulate birth rate and reduce death rate as well as to resettle compatriots from abroad bring quick desirable effect.

3. The rates of gross output and GDP growth are taken to be equal.

4. The sectoral coefficients of labour-intensiveness, capital-intensiveness and per unit consumption of material are determined endogenously on the basis of calculated gross output and size of labour resources.

The following scenarios of the development of the Russian economy were studied.

The following assumptions were made in the first (optimistic) scenario.

1. On the basis of the hypothesis that GPD in Russia will double in ten years and the living standard of the population will grow correspondingly it was concluded that the growth rate of expenditure on end consumption in 2008 - 2012 should be equal to at least 41%.

In this scenario it is assumed that the economy of Russia follows the innovational way of development, which makes it possible to provide a considerable growth of the living standard of the population, make it possible to diversify export (get rid of dependence on world market prices on energy carriers) and to provide a more stable economic growth.

2. During the whole projected period the input of fixed assets will grow at least 90%. The rate of investments into fixed assets will at least double.

3. During the whole forecasting period retirement compensation rates for the active and passive parts of fixed assets in the Russian economy as a whole will grow 3 and 1.5 times accordingly. The scenario of a considerable renewal of the production mechanism makes it possible to accelerate the introduction of new technologies into production processes and raise the efficiency of production.

The second (pessimistic) scenario is based on the following assumptions.

The GDP and gross output of the Russian economy in 2008 – 2012 will grow at the rate of approximately 30% that corresponds to the average annual growth of 5.2%. This rate conforms to the inertial scenario of the development of the economy of the Russian Federation included into the forecast of the Ministry of Economic Development and Trade of Russia for the period 2008 – 2012 [9].

2. The scenario assumes a slower replacement of machines and equipment. The retirement compensation rate for the active part of fixed assets will grow gradually from 1.6% in 2007 to 3.2% in 2012; for the passive part of fixed assets it will increase from 1% to 1.2% correspondingly (see table 4). Fixed capital investments will grow approximately 38% (97% in the optimistic scenario), while investments into the active part of fixed assets will increase by 48% (214% in the optimistic scenario).

As we can see from the above brief description of the hypotheses that underlie the estimates for different scenarios, they investigate different versions of accelerated renewal of fixed assets in the Russian economy, primarily their active part.

It is important to note that during the period under review, within the frameworks of the first and second projections, there occurs a considerable growth of production in capital-building engineering industry (see Appendix). At the same time it is assumed that in all the industries of the economy the utilization of fixed assets will become more intensive: there will be growth of capital productivity or decrease of capital intensiveness (see table 4). The growth of capital productivity in the forecasting estimates is explained by introducing into production, as it expands, of a considerable part of production capacities not utilized at present and by using new more effective fixed assets that are put into operation during the investigated period.

An important condition for reaching the rates of production growth assumed in the scenarios is the growth of fixed capital investments; with priority investments into capital-building and adjacent industries (see Appendix).

Let us note another factor limiting the economic growth of Russia: lack of skilled workforce. In recent thirteen years, the system of replenishment of qualified labour resources has experienced negative changes, especially in the area of training skilled workers. That is why further growth of production in the majority of industries will be inevitably hindered by the problem of providing qualified staff.

Index	2008	2009	2010	2011	2012	2012/2007		
Total Gross Output and GDP								
Optimistic Scenario	107,5	107,5	107,5	107,5	107,5	143,6		
Pessimistic Scenario	105,4	105,4	105,4	105,4	105,4	130,1		
Gross output of the 1 st subdivision								
Optimistic Scenario	109,2	106,9	107,0	107,4	107,6	144,4		
Pessimistic Scenario	106,8	104,6	104,6	104,9	105,0	128,7		
Gross output of the 2nd subdivision								
Optimistic Scenario	104,4	108,6	108,5	107,7	107,2	142,0		
Pessimistic Scenario	102,8	106,8	107,0	106,4	106,1	132,7		
Total Fixed Assets								
Optimistic Scenario	101,6	101,8	102,0	102,3	102,7	110,8		
Pessimistic Scenario	101,5	101,6	101,6	101,7	101,8	108,6		
Including								
Active Part of Fixed Assets								
(machines and equipment)								
Optimistic Scenario	102,0	102,0	102,1	102,4	102,7	111,8		
Continuation of Table 2.8								

	Table 3.	Projected	growth	rates o	f certain	major	indices	of the	Russian	economy	in /
2008 -	2012, %.										

Pessimistic Scenario	102,0	102,0	102,0	102,0	102,0	110,3
Passive Part of Fixed Assets						
(structures)						
Optimistic Scenario	101,4	101,6	101,9	102,3	102,6	110,3
Pessimistic Scenario	101,3	101,4	101,5	101,6	101,7	107,6
Total Fixed Capital Investments						
Optimistic Scenario	114,6	113,9	114,4	114,7	115,0	196,9
Pessimistic Scenario	106,7	106,5	106,6	106,8	106,9	138,3
Including						
Investments into the Machines and Equipment						
Optimistic Scenario	117,5	115,9	116,2	116,3	116,2	213,7
Pessimistic Scenario	109,1	108,0	108,1	108,0	107,9	148,4
Investments into the Structures						
Optimistic Scenario	112,3	112,3	112,8	113,4	113,9	183,8
Pessimistic Scenario	104,9	105,3	105,4	105,7	106,0	130,4
Total Fixed Assets Retirement Compensation Rate, %						Retirement Compensation Rate
						ın 2007, %
Optimistic Scenario	1,5	1,8	2,2	2,5	2,8	<u>in 2007, %</u> 1,2
Optimistic Scenario Pessimistic Scenario	1,5 1,4	1,8 1,5	2,2 1,7	2,5 1,8	2,8 2,0	1,2 1,2
Optimistic Scenario Pessimistic Scenario Including	1,5 1,4	1,8 1,5	2,2 1,7	2,5 1,8	2,8 2,0	1n 2007, % 1,2 1,2
Optimistic Scenario Pessimistic Scenario Including Retirement Compensation Rate of the Machines and Equipment, %	1,5 1,4	1,8 1,5	2,2	2,5 1,8	2,8 2,0	1n 2007, % 1,2 1,2
Optimistic Scenario Pessimistic Scenario Including Retirement Compensation Rate of the Machines and Equipment, % Optimistic Scenario	1,5 1,4 2,3	1,8 1,5 2,9	2,2 1,7 3,6	2,5 1,8 4,2	2,8 2,0 4,9	1,2 1,2 1,2 1,6
Optimistic Scenario Pessimistic Scenario Including Retirement Compensation Rate of the Machines and Equipment, % Optimistic Scenario Pessimistic Scenario	1,5 1,4 2,3 1,9	1,8 1,5 2,9 2,3	2,2 1,7 3,6 2,6	2,5 1,8 4,2 2,9	2,8 2,0 4,9 3,3	1n 2007, % 1,2 1,2 1,2 1,6 1,6
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Optimistic ScenarioPessimistic ScenarioIncludingRetirement Compensation Rateof the Machines and Equipment,%Optimistic ScenarioPessimistic ScenarioRetirement Compensation Rateof the Structures, %Optimistic ScenarioPessimistic ScenarioPessimistic ScenarioOptimistic ScenarioPessimistic ScenarioOptimistic ScenarioPessimistic ScenarioOptimistic ScenarioPessimistic ScenarioOptimistic Scenario	1,5 1,4 2,3 1,9 1,1 1,0 94,5 96,3 98,0	1,8 1,5 2,9 2,3 1,2 1,1 94,7 96,4 98,0	2,2 1,7 3,6 2,6 1,3 1,1 94,9 96,4 98,3	2,5 1,8 4,2 2,9 1,4 1,2 95,2 96,5 98,3	2,8 2,0 4,9 3,3 1,5 1,2 95,5 96,6 98,4	1n 2007, % 1,2 1,2 1,2 1,6 1,6 1,6 1,0 1,0 1,0 77,2 83,5 91,4

Note: results of estimates for the Russian economy are based on the Dynamic Input-Output Model

2. Results and interpretation of forecasting estimates based on the Dynamic Input-Output Model with fuzzy parameters

The second stage of projecting the development of the Russian economy consisted in making estimates with the help of the Dynamic Input-Output Model with fuzzy parameters. These estimates were based on the results of projecting the economy of Russia using a Deterministic Input-Output Model.

In the course of the estimates, several experiments were carried out. The degree of plausibility of a particular assumption was understood in the following way (see [16]).

Namely, fuzzy set *A* in space *X* is understood as a geometrical object having the following property: for each $x \in X$, number $\chi_A(x) : 0 \le \chi_A(x) \le 1$ is determined that is interpreted as the degree of plausibility of the statement that $x \in A$.

If $\chi_A(x) = 0$, statement $x \in A$ is absolutely implausible; if $\chi_A(x) = 1$, then statement $x \in A$ is absolutely plausible. Function $\chi_A : X \to I$ is called the function of membership of a (fuzzy) set A. Here I = [0,1], I^X - is the domain of measurable images $f : X \to I$.

In fact, a fuzzy assignment of parameters in the Dynamic Input-Output Model and the computation of fuzzy values `of economic indices lead to a new understanding of macroeconomic stability. The methodology of assessing the reliability of forecasted indices proposed in this work [17] can be also interpreted as the assessment of stability of computed fuzzy indices with respect to a fuzzy description of model parameters.

In order to build the membership function of computed indices a stochastic algorithm described in the paper [17] was applied. In each experiment, particular parameters of the Dynamic Input-Output Model were assigned within the framework of particular constraints indistinctly; 200 estimates based on of the Dynamic Input-Output Model were carried out. Their results, in terms of studied parameters, were processed with the help of a stochastic procedure of building membership function and are illustrated with the help of the diagrams below. For instance, figure 1 demonstrates a fuzzy representation of a projected growth rate of gross output of the Russian economy between 2008 and 2012.

In the estimates given below, the degree of fuzziness of parameters varied within the limits of 10 to 25 per cent from the values of parameters defined on the basis of retrospective information analysis.

In order to estimate the level of stability each selected index (for instance, gross output of the national economy) was compared with a standard fuzzy description of its most probable value that was calculated on sample basis. In order to calculate the most probable value x_0 the problem of maximizing the function of membership χ_p of index *p* was solved:

 $\chi_p(x_0) = \max_{x \in R} \chi_p(x),$

1.

where R is a real line.

A standard fuzzy presentation of value x_0 was calculated with the help of a stochastic procedure where the degree of fuzziness was equal to half the amplitude of sampling of a studied parameter, with the degree of fuzziness of variable parameters equal to 10%.

Let us briefly describe some results of experimental estimates.

It was determined whether an event was probable, that is whether the growth rate of gross output of the Russian economy would be equal to 143.6% (the most probable index value in the optimistic scenario) in the conditions when the most important parameters of the Dynamic Input-Output Model are presented indistinctly, within the limits specified earlier. The probability degree of achieving the growth rates of gross output of the 1^{st} (144.4%) and 2^{nd} (142.0%) subdivisions was determined in the same way. At the same time, the following parameters were set in an indistinct way: 1) the growth rates of employment in the economy during five years; 2) the value of fixed assets put into service in each year of the projected period; 3) the growth rate of net export during five years; 4) the growth rate of each element of materials output ratio matrix during five years; 5) the sectoral structure of fixed assets put into service; 6) labour productivity growth rate (the rate of growth of each element of the vector was specified in a fuzzy way). "The excitation" of the parameters enumerated above was conducted within a wide range values; here we mean their deviation from the projected value determined through expertise by analyzing retrospective data: \pm

10%, \pm 15%, \pm 20%, \pm 25%. The deviation of the variable parameters grew from year to year, which complied with the hypothesis that fuzziness in their values increases as we move away from the base year. For example, for the option of parameters variation by \pm 10%, the dynamics of deviations growth was given as: \pm 2% in 2008, \pm 4% in 2009, \pm 6% in 2010, \pm 8% in 2011, \pm 10% in 2012. The growing deviations dynamics for other ranges of parameters deviations were assigned in a similar way. Examples of the results of estimates in the first experiment are given in Fig. 1 and 2. The results of the first experiment are summarized in Table 4.

Table 4. Dependence of the gross output stability level of the Russian economy on the degree of variation of 6 groups of parameters

Index	± 10%	± 15%	± 20%	± 25%
Stability level for gross output as a whole	95,6	79,1	69,0	50,5
Stability level for gross output of the 1 st subdivision	94,8	77,0	63,1	49,2
Stability level for gross output of the 2 nd subdivision	93,9	74,1	61,6	47,4

From the results of the calculations given above a conclusion can be drawn that the gross output of the second subdivision of the Russian economy appeared the least stable in the case of variations in 6 major parameters.

Fig 1. Membership function of a fuzzy gross output growth rate of the Russian economy according to the optimistic development scenario, with a fuzzy assignment of 6 major parameters under the variations range of \pm 10%.

The level of stability is equal to 95,6% (plausibility degree of coincidence of sample and standard indices).



Fig 2. Membership function of a fuzzy gross output growth rate of the Russian economy according to the pessimistic development scenario, with a fuzzy assignment of 6 major parameters with the variations range of \pm 15%.

The level of stability is equal to 79,1% (plausibility degree of coincidence of sample and standard indices).



2. It was determined whether an event was probable, that is whether the growth rate of gross output of the Russian economy would be equal to 143.6% (the most probable index value in the optimistic scenario) in the conditions when the size of fixed assets put into operation and labour productivity in the Dynamic Input-Output Model are presented indistinctly within the limits specified earlier. "The excitation" of the parameters grew from year to year as in the first experiment. Cases of a fuzzy input of fixed assets and labour productivity were studied when these parameters deviated from the standard value by \pm 10% and \pm 20%. Similarly, the probability degree of achieving the growth rates of gross output of the first (144,4%) and second (142,0%) subdivisions was determined. The results of the second experiment are summarized in Table 5.

Table 5. Dependence of the gross output stability level of the Russian economy on the degree of variation in the description of fixed assets put in service and labour productivity

Index	± 10%	± 20%
Stability level for total gross output	93,9	56,0
Stability level for gross output of the 1 st subdivision	95,0	68,1
Stability level for gross output of the 2 nd subdivision	97,2	38,2

From the data given in Table 5 it is seen that the economy of Russia is very sensitive to changes in the fixed assets put in service and fluctuations in labour productivity connected with them. An enormous instability in the dynamics of the output of the 2^{nd} subdivision against the fluctuations of variable parameters deserves special attention.

Based on the preliminary forecasting estimates using the Dynamic Input-Output Model with fuzzy parameters, we can draw a conclusion that *sustainable economic growth that provides a considerable improvement of the living standard of the population is possible only in the conditions of a stable renewal of fixed assets through providing high rates of introducing fixed assetsput in service leading to a considerable growth of labour productivity.*

3. The third experiment studied the stability of capital gross output ratio of the Russian economy against fluctuations in the fixed assets put in service and, consequently, changes in labour productivity.

The results of this experiment are illustrated in Fig3 - 4

Fig 3. Membership function of a fuzzy growth rate of capital gross output ratio of the Russian economy, according to the optimistic scenario of its development, with a fuzzy assignment of the dynamics of fixed assets put into service and labour productivity under the variations range of \pm 10%. The level of stability is equal to 93,4 % (plausibility degree of coincidence of sample and standard indices).



The results given in Graphs 3 - 4 testify that capital intensiveness is very sensitive (highly unstable) to variations in the input of fixed assets and labour productivity.

Fig 4. Membership function of a fuzzy growth rate of capital gross output ratio of the Russian economy, according to the optimistic scenario of its development, with a fuzzy assignment of the dynamics of fixed assets put into service and labour productivity under the variations range of \pm 20%. The level of stability is equal to 61.5 % (plausibility degree of coincidence of sample and standard indices).



4. The next experiment consisted in the "excitation" of the rates of growth of materials output ratio. During five years, the growth rate of each element of materials output ratio matrix varied within the range of \pm 10% μ \pm 20%. The results of estimates are presented in graphs 5 – 8.

Fig 5. Membership function of a fuzzy growth rate of gross output of the first subdivision of the Russian economy, according to the optimistic scenario of its development, with an fuzzy assignment of the dynamics of materials output ratio under the variations range of \pm 10%. The level of stability is equal to 88,7 % (plausibility degree of coincidence of sample and standard indices).



Fig 6. Membership function of a fuzzy growth rate of gross output of the first subdivision of the Russian economy, according to the optimistic scenario of its development, with an fuzzy assignment of the dynamics of materials output ratio under the variations range of \pm 20%. The level of stability is equal to 69.0 % (plausibility degree of coincidence of sample and standard indices).



Fig 7. Membership function of a fuzzy growth rate of gross output of the second subdivision of the Russian economy, according to the optimistic scenario of its development, with a fuzzy assignment of the dynamics of materials output ratio under the variations range of \pm 10%. The level of stability is equal to 89,5 % (plausibility degree of coincidence of sample and standard indices).



Fig 8. Membership function of a fuzzy growth rate of gross output of the second subdivision of the Russian economy, according to the optimistic scenario of its development, with a fuzzy assignment of the dynamics of materials output ratio under the variations range of \pm 20%. The level of stability is equal to 88,8 % (plausibility degree of coincidence of sample and standard indices).



The results of the fourth experiment demonstrated a much greater stability of the second subdivision in comparison with the first one in the conditions of varying rates of growth of materials output ratio.

Basic Conclusions

- 1. For the Russian economy to join the path of stable economic growth with a considerable (of at least two times) growth of GDP in 10 years (2008-2017) it is necessary to increase investments into fixed assets approximately two times in 2008-2012, including investments into the machines and equipment of at least 2.1 2.2 times.
- 2. The quantitative assessment of parameters of a rapid renewal of fixed assets show that the retirement compensation rate of fixed assets should grow from 1.2% in 2007 to 2.8% in 2012, while for the active part of fixed assets (machines and equipment) it should increase from 1.6 in 2007 to 4.9 in 2012.
- 3. The results of estimates based on the Dynamic Input-Output Model show that in the period 2008 2012 the gross output of asset-building sectors of the engineering industry and construction should grow at the rate of approximately 210 % and 180 % correspondingly. If it is not possible to provide such growth rates of asset-building sectors, there should be a considerable growth of import of machines and equipment, which will have negative consequences for balancing the balance of payments.
- 4. A fuzzy assignment of parameters for the Dynamic Input-Output Model and the computation of fuzzy values of projected indices can be interpreted as the assessment of stability of the computed fuzzy indices (gross output, fixed assets, etc.) in the conditions of a fuzzy description of model parameters.
- 5. A fuzzy description of growth rates of materials output ratio with different degrees of their "excitation" demonstrated a much greater resistance of the second subdivision to variations of this index in comparison with the first one.
- 6. A fuzzy description of the size of fixed assets put into service and labour productivity in the Dynamic Input-Output Model showed that in the projected period the economy of Russia demonstrates high instability if these parameters vary. It implies that sustainable economic growth that ensures a considerable improvement of the living standard of the population is possible only in the conditions of a stable renewal of fixed assets by means of providing high rates of input of fixed assets leading to a marked increase of labour productivity.

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APPENDIX

Optimization Intersectoral Dynamic Model with Fuzzy Parameters

A distinctive feature of constructing the Dynamic Input-Output Model is splitting the production sphere into two subdivisions. In accordance with the methodology of national accounts, the sphere of production includes both material and non-material production, as well as, partly, housekeeping. Thus, the first subdivision of the gross output production sphere comprises the production of the means of production and services (both material and non-material ones) included into intermediate consumption. The second subdivision consists of the production of commodities and services (both material and non-material ones) involved into final consumption. Such interpretation of the two subdivisions of the national economy is explained in the paper [4].

The model uses the following parameters that are described in terms of fuzzy sets.

n – number of sectors in the economy;

m - number of sectors in the first subdivision $(m \le n)$;

k – number of asset-building sectors ;

T – number of forecasting time periods;

l – number of labour resources types specified in the model;

 $a_{ij}(t)$ - ratios of direct material costs of sector *i* per unit of production *j* in the *t* time period;

 $c_{hj}(t)$ - ratios of labour intensiveness of a sector *j* for the *h* type of labour resources in the *t* time period;

 $b_{ij}(t)$ - ratios of capital intensiveness of a sector *j* for the *i* -type of fixed assets in the *t* time period;

 ϑ_{ij} - construction lag in sector *j* for the *i* type of fixed assets;

 $k_{ij}(t, \tau)$ - retirement rate of fixed assets of *i* type in sector *j* aged τ in the *t* time period;

 $B_{ij}(t,\tau)$ - fixed assets of *i* type in sector *j* put in service in the *t* time period ;

 $K_{ij}(t,t+\tau)$ - investments of type *i* in sector *j* in year *t* into the facilities put into operation in $t+\tau$ time period;

 $K_{ij}^{*}(t)$ - total investments of type *i* in sector *j* in the *t* time period;

 $\mu_{ij}(t,\tau)$ - ratio showing which part of fixed assets input in sector *j* in time period $t + \tau$ is formed due to investments of type *i* in the *t* time period so as

$$\mu_{ij}(t,\tau) = K_{ij}(t,t+\tau) / \left(\sum_{i=1}^{k} B_{ij}(t+\tau) \right);$$

 $L_h(t)$ - size of *h*- type of labour resources that can potentially be employed in the economy in *t* time period;

 $F_{ij}(t, t - \tau)$ - fixed assets of *i* - type of sector *j* introduced in the period $t - \tau$ by the end of year *t*,

 $F_{ij}^{*}(t)$ - fixed assets of *i* – type of sector *j* by the end of time period *t*;

 $N_{ij}(t)$ - construction-un-progress of fixed assets of type *i* in sector *j* by the end of time period *t*;

 $f_j(t)$ - weighting coefficients of *j* production sector in the target functional of the economic system.

It is assumed that all the introduced parameters are fuzzy multitudes.

Let the economic system embrace n sectors, where $1 \le i \le k$ are asset-building ectors, $k < i \le m$ are not-asset-building sectors of the first subdivision, and $m < i \le n$ are the sectors of the second subdivision.

As the model parameters are fuzzy multitudes, all further arithmetic transformations done in accordance with the rules of fuzzy arithmetic will also constitute fuzzy sets. Let us mark a fuzzy produced gross output as $x_j(t)$, and a fuzzy utilized gross output as $\bar{x}_j(t)$ of sector *j*, a fuzzy net export of *j* product as $S_j(t)$, a fuzzy growth of reserves as $\Delta z_j(t)$ and fuzzy losses of output as $\Pi_j(t)$ of sector *j* during *t* period of time. By analogy with [18], the equation of product balance of sector *j* will be presented as :

$$x_{i}(t) = \bar{x}_{i}(t) + S_{i}(t) + \Delta z_{i}(t) + \Pi_{i}(t)$$
(1)

The reproduction of fixed assets in the model of dynamic intersectoral balance with lags is described as the process of exchange of a utilized output of asset-building sectors of t period for the introduction of fixed assets of t period that is mediated by the change in the volume of construction in progress.

The application of lag indices makes it possible to tie the process of output production by asset-building sectors of machine-building and construction as well as export and import of the output of these sectors in each time period with preceding and subsequent periods. Part of the output produced by asset-building sectors of the economic system in each period ensures the continuation of construction begun before and part of it is exported. It provides the connectedness of investments and, consequently, the dependence of their volume, industrial and technological structure on the previous investments and on the size of import of asset-building sectors output. In each period of time, the fixed assets input differs in its physical composition due to the output of machine-building and construction industries utilized during preceding and present periods. During t period, construction-in-progress of j sector $(1 \le j \le n)$ receives the output of asset-building sector i in volume $K_{ij}^*(t)$, which is distributed among different layers of construction-in-progress. Investments are determined by the formula

$$K_{ij}^{*}(t) = \sum_{u \ge 0} K_{ij}(t, t+u) .$$
⁽²⁾

The input of fixed assets $B_{ij}(t)$ in sector *j* during *t* time period is formed from the used product of *i* asset-building sector according to the formula

$$B_{ij}(t) = \sum_{u \ge 0} K_{ij}(t - u, t)$$
 (3)

The size of investments $K_{ij}(t,t+u)$ into the layer of construction-in-progress introduced during t+u time period is calculated through the input of fixed assets in this period by the formula

$$K_{ij}(t,t+u) = \mu_{ij}(t,u) \cdot \sum_{i=1}^{k} B_{ij}(t+\tau) .$$
(4)

Coefficients $\mu_{ij}(t,u)$ are the integral characteristic of fixed assets input and depend on the technology and intensiveness of construction in sector *j*. At the same time, the technology of construction consists of a finite number of stages. Then investments are determined through the formula

$$K_{ij}(t,t+u) = \sum_{\nu} \left(\xi_j(t,t+u,\nu) \cdot \eta_{ij}(t+u,\nu) \cdot \sum_{i=1}^k B_{ij}(t+\tau) \right), \qquad (5)$$

where $\eta_{ij}(t+u,v)$ is the share of the input of fixed assets of *i-type* in branch *j* in *t+u* time period that is formed during *v* stage of construction; $\xi_j(t,t+u,v)$ is part of stage *v* performed during *t* time period (*u* periods before the introduction of the present layer). Depending on the size of expected investments into fixed assets several successive stages can be performed during one period or one stage that can last several periods. Formulae (4), (5) are the basic ones for determining the size of investments into fixed assets. Additional control parameters $\xi_j(t,t+u,v)$ in formula (5) make it possible to forecast coordinated input of fixed assets and fixed capital investments in the conditions of changing construction periods. For this purpose, standards $\xi_i(t,t+u,v)$ take into account the acceleration or deceleration of capital construction rates.

Recurrent ratios for re-computing construction-in-progress are described by the formula

$$N_{ij}(t) = N_{ij}(t-1) - \sum_{u=1}^{\vartheta_{ij}-1} K_{ij}(t-u,t) + \sum_{u=1}^{\vartheta_{ij}-1} K_{ij}(t,t+u).$$
(6)

Recurrent ratios for determining the size of fixed assts of i type in branch j aged u by the end of time period t are specified by the formula

$$F_{ij}(t,0) = B_{ij}(t), \quad F_{ij}(t,u) = F_{ij}(t-1,u-1) \cdot (1-\kappa_{ij}(t,u)).$$
(7)

The model of fixed assets reproduction (2)-(7) is used for evaluating fixed capital investments and their technological structure by sectors through the anticipated introduction of fixed assets adjusted for the construction lag and $\xi_j(t,t+u,v)$ operation regime of investment package.

The gross output $x_j(t)$ of *j* asset-building sector during *t* time period is determined by the formula

$$x_{j}(t) = \sum_{j=1}^{n} K_{ij}^{*}(t) + S_{i}(t) + n_{i}(t)$$
(8)

The balance between production and utilization of the output of non-asset-building sectors of the first subdivision looks as follows

$$x_{j}(t) = \sum_{j=1}^{n} a_{ij}(t) \cdot x_{j}(t) + S_{i}(t) + n_{i}(t), \quad k < i \le m.$$
(9)

Correlations for forming the output of the second subdivision sectors are presented as

$$x_{i}(t) = Q_{i}(x_{i}(t-1), S_{i}(t-1), \lambda, t) + S_{i}(t), \quad m < i \le n,$$
(10)

where Q_i is images synthesizing the structure and dynamics of needs (normally these are monotonously growing functions of λ parameter.)

Labour resources limits are described by the system of inequalities

$$\sum_{j=1}^{n} c_{hj}(t) \cdot x_{j}(t) \le L_{h}(t), \quad h = 1, ..., l$$
(11)

Fixed assets constraints are described by the system of inequalities

$$b_{ij}(t) \cdot x_j(t) \le F_{ij}(t), \quad 1 \le i \le k, \quad 1 \le j \le n$$
(12)

Let Ω be the set of fuzzy paths of the development of the economic system $x_j(t)$ that complies with constraints (2)-(3), (5)-(12) in each time period *t* and let us formulate the problem of fuzzy optimization

$$\sum_{t=1}^{T} \sum_{j=1}^{n} f_j(t) \cdot x_j(t) \Rightarrow max, \quad x \in \Omega , \qquad (13)$$

where the set of possible paths Ω and the coefficients of the maximized function $f_j(t)$ are fuzzy.

The solution of the problem (13) under the input of fixed assets $B_{ij}(t)$, labour resources $L_k(t)$, as well as standards $\eta_{ij}(t+u,v)$, $\xi_j(t,t+u,v)$, $\kappa_{ij}(t,u)$, $a_{ij}(t)$, $S_i(t)$, $c_j(t)$ for each time period from [0;*T*], gives a fuzzy system of development indices of the economic system, including gross output $x_j(t)$, fixed capital investments $K_{ij}^*(t)$, input of fixed assets $B_{ij}(t)$ and fixed assets by the end of each time period $F_{ij}^*(t) = \sum_{u \ge 0} F_{ij}(t,u)$.